

# The Top and QCD

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## Topics:

- ❖ Top-pair: total cross-section
- ❖ PDF's and their impact on top measurements
- ❖ Top-mass
- ❖ Forward-Backward asymmetry

# Top-pair cross-section

# Current status

Top-pair cross-section, 20 years later:  
The state of the art is **still** NLO QCD corrections ☺

Nason, Dawson, Ellis (1988-90)  
Beenakker, Kuijf, van Neerven, Smith (1989)  
Beenakker, van Neerven, Meng, Schuler, Smith (91)  
Mangano, Nason, Ridolfi (1992)  
Bernreuther et al. (2004)  
M. Czakon, A.M. (2008)

- The only improvement over 20 years: now we know it analytically.
- Such slow progress is for a good reason:  
top is very hard to calculate!
- Theoretical uncertainties are not as small as we would like them to be:

NLO corrections 50%

NLO uncertainty 10% (more details to follow).

Can we get the uncertainties down to few percent?

# What to do?

- ❖ Clearly, the best way is to just calculate the NNLO corrections.

This is very complicated! The complexity is  $\sim$  3-loop massive box !!

The best strategy is known, and people are working hard on this:

M. Czakon and A. M.

Ingredients for the two-loop amplitudes already exist:

- 2-loop  $qq \rightarrow QQ$  amplitude (numerically, high precision)

M. Czakon

- all 2-loop amplitudes in the limit  $m_{\text{top}} \rightarrow 0$  (analytically)

M. Czakon, A.M. , S. Moch

- 2-loop  $gg \rightarrow QQ$  amplitude (numerically) – **expect to appear this year** 😊

- One loop amplitudes squared known too.

Knierl et al; Anastasiou, Aybat

Bottleneck: IR subtraction scheme! (recall:  $e^+e^- \rightarrow 3$  jets)

A. Gehrmann-De Ridder et al;  
S. Weinzierl

# What to do?

❖ Second approach: soft gluon (threshold) resummation.

The only source of new information in top production in the last > 10 years

Developed (NLL): Stermann et al mid-90's

Bonciani, Catani, Mangano, Nason '98

Applied (NLL):

Kidonakis, Laenen, Moch, Vogt;

Cacciari et al, Moch Uwer, Czakon AM

How much can resummation tell us ?

Past analyses NLO/NLL show that it brings small reduction in the theoretical uncertainties, i.e. 12-15% down to say 10%.

Cacciari et al  
Kidonakis, Vogt;

# From NNLL to NNLO ??

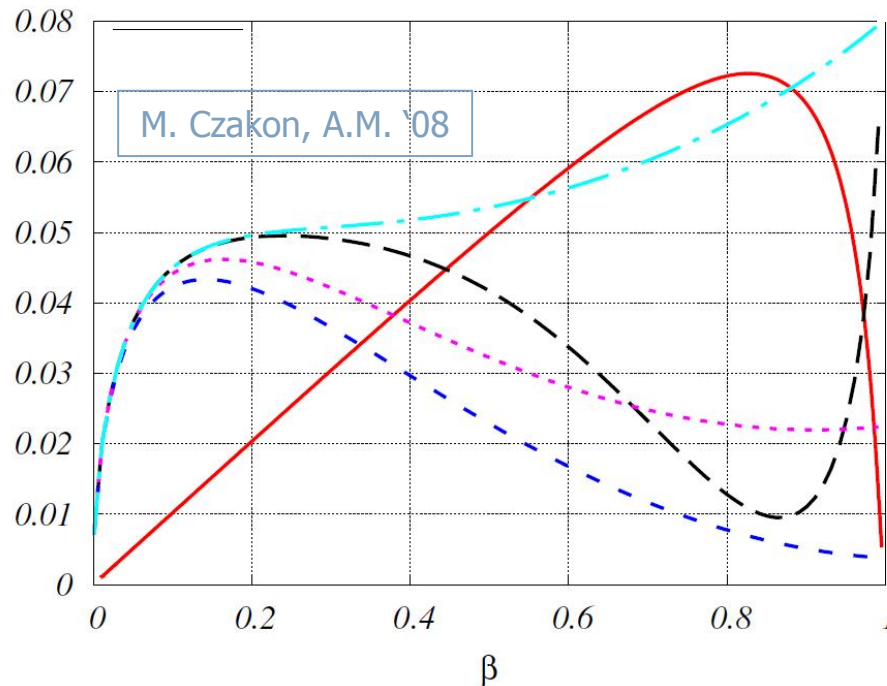
- ❖ Soft gluon resummation can predict **some** terms at NNLO
- ❖ Typically, this contains **very limited** information about NNLO
- ❖ Such approach is based on the following assumptions:
  - ✓ soft approximation is dominant (**incorrect even at NLO, see next slide**),
  - ✓ partonic flux samples the threshold region;  
that additionally enhances the soft terms (**incorrect, see next slide**),
  - ✓ NNLL resummation (**not yet possible**)
    - In fact the NLO/NLL (one-loop) result was completed just 7 months ago! Czakon, A.M. '08
    - First partial results needed for NNLL just appeared: A.M., Sterman, Sung '09  
Becher, Neubert '09  
Kidonakis '09
    - Work for NNLL underway!

# The NLO partonic cross-section and flux

The observed cross-section is an integral over the product of:

- Partonic cross-section (NLO),
- Partonic flux (incl. Jacobian).

$$\sigma(s_{\text{had}}) = \sum_{ij} \int_0^{\beta_{\text{max}}} d\beta \Phi(\beta) \hat{\sigma}_{\text{part}}(\beta)$$



$$\rho = \frac{4m_t^2}{s}$$

$$\beta = \sqrt{1 - \rho}$$

$$\eta = \frac{s}{4m_t^2} - 1$$

- ❖ The soft approximation is not a good approximation to FO (at NLO)
- ❖ The flux does not predominantly sample the threshold region!
- ❖ Sub-leading power terms large!

Noticed first by Bonciani, Catani, Mangano, Nason '98

# Top quark pair: “the numbers”

The central values (LHC @ 14 TeV):

- FO NLO / FO LO: 50%
- NLL / FO NLO: 4% (circa early 2008)
- Some beyond NLL effects / FO NLO: 0.8% Moch, Uwer '08
- New NLO effects / FO NLO: 1~1.5% Czakon, AM '08

**Important:** No genuine NNLO term is known (could easily give 5% shift) !

**Lesson:** in top production, large contributions come from hard, not soft emissions

Perhaps also relevant for the FB asymmetry ?

# Top quark pair: central values; PDF's

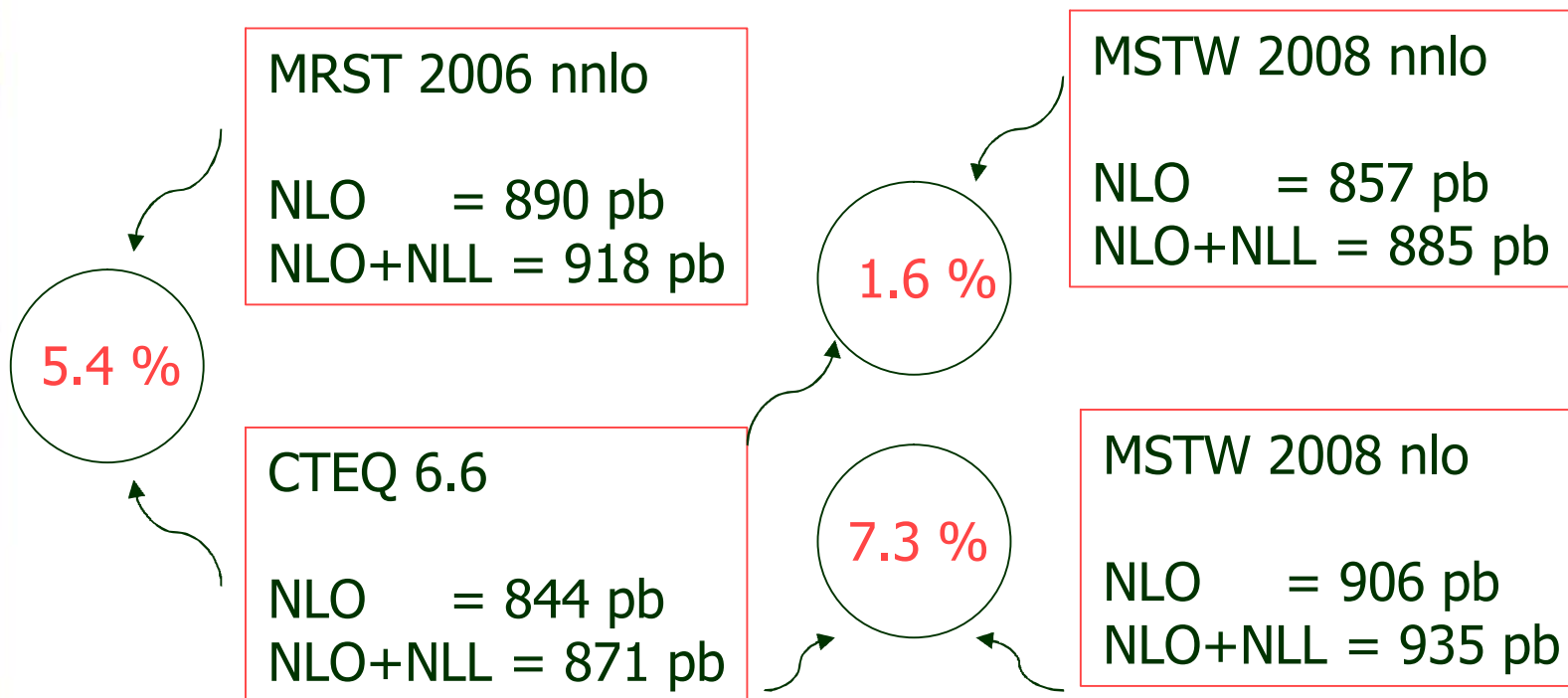
Comparison of central values for:

Czakon, AM in progress

- $m_{\text{top}} = 172.4 \text{ GeV}$
- $\mu = m$
- correct exact hard matching coefficients
- Coulombic effects not elaborated upon.

$\alpha_s(M_Z)$  :

CTEQ 6.6: 0.118  
MRST 2006 nnlo: 0.119  
MSTW 2008 nnlo: 0.117  
MSTW 2008 nlo: 0.120



# Top quark pair: PDF's

CTEQ 6.6

NLO = 844 pb  
NLO+NLL = 871 pb

7.3 %



MSTW 2008 nlo

NLO = 906 pb  
NLO+NLL = 935 pb

- At NLO the two sets predict 7% difference in central values
- Inconsistent with expectation for 3% uncertainty due to each set
- NOTE: these are NLO sets; at NLO everything (regarding PDF) is well understood and sufficiently well known.
- Perhaps the PDF uncertainties are much larger than thought?
- What about NNLO PDF sets then?

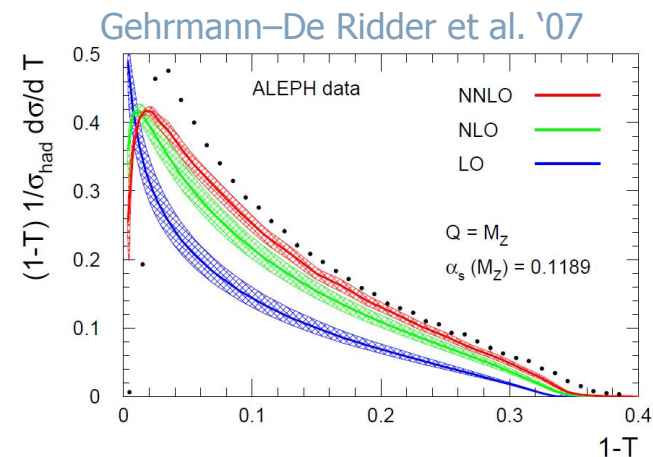
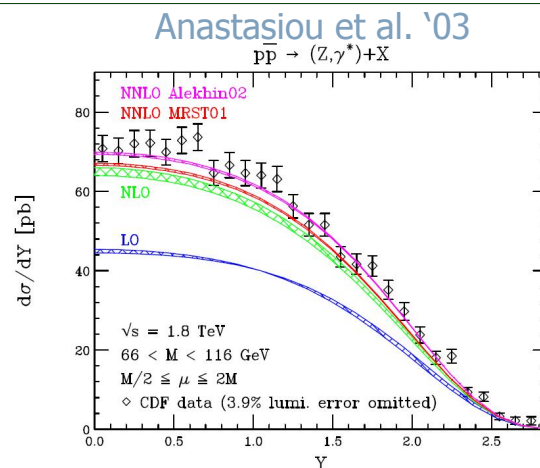
# Top quark pair: “the uncertainty”

Current theory error estimate (NLO/NLL):  $\sim 10\%$

- 1) 3% uncertainty would be just half the diff. between the NLO PDF sets (7%)
- 2) Scale variation is not a true error estimate in t-tbar (or anywhere else ☺)
  - ✓ Accidental cancelation between renormalization/factorization scales
  - ✓ Large sub-leading terms
  - ✓ Likely large NNLO corrections
- 3) No genuine NNLO term is known; could easily shift  $\sigma_{TOT}$  by 5% !

Catani et al.

2 NNLO examples  
of  
underestimated  
error by standard  
scale variation:  
 $M/2 < \mu < 2M$



# Top quark pair: open problems

- ❖ We do not measure top quarks, but their decay products
- ❖ Beyond LO, theorists do not have that much to offer here ☹
- ❖ So far all approaches based on neglecting production/decay interference. Likely small effect.
- ❖ MC@NLO: NLO production + LO decay +shower
- ❖ top pair in MCFM?
- ❖ With new unitarity-based methods:  
interesting progress reported at Loopfest '09 by [Melnikov and Schulze](#)
- ❖ Speed at NLO likely to be an issue.

# Top quark mass

# Top mass: why we care?

We need to know the top mass because it is “portable”:

Places where the top mass is crucial:

- Higgs mass

*Precision Electroweak Measurements and Constraints on the Standard Model* [arXiv:0811.4682v1 \[hep-ex\]](https://arxiv.org/abs/0811.4682v1)

➤ Lower limit from direct searches:

ALEPH, DELPHI, L3, and OPAL Collaboration '03

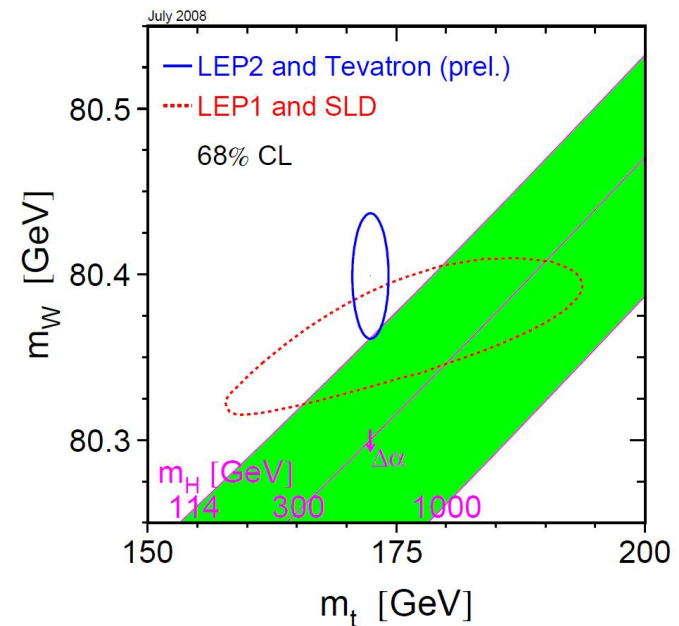
$M_H > 114$  GeV; recent exclusion of 160-170 GeV range from Tevatron

➤ Indirect constraints from LEP +  $M_{\text{top}}$  +  $M_W$

$M_H = 84 + 34 - 26$  GeV

$$M_t = 173.1 \pm 1.3 \text{ GeV}/c^2$$

Current best measurement  
CDF+D0: 0903.2503



# Top mass: why we care?

Places where the top mass is crucial:

- Higgs-inflation

Bezrukov, Shaposhnikov '07-'08  
De Simone, Hertzberg, Wilczek'08

Assume non-minimal coupling to gravity:

$$\mathcal{L}_h = -|\partial H|^2 + \mu^2 H^\dagger H - \lambda(H^\dagger H)^2 + \xi H^\dagger H \mathcal{R}$$

Then: **Higgs = inflaton** provided:

1)  $10^3 < \xi < 10^4$

2)  $m_h > 125.7 \text{ GeV} + 3.8 \text{ GeV} \left( \frac{m_t - 171 \text{ GeV}}{2 \text{ GeV}} \right) - 1.4 \text{ GeV} \left( \frac{\alpha_s(m_Z) - 0.1176}{0.0020} \right) \pm \delta$

3)  $m_h \lesssim 190 \text{ GeV}$

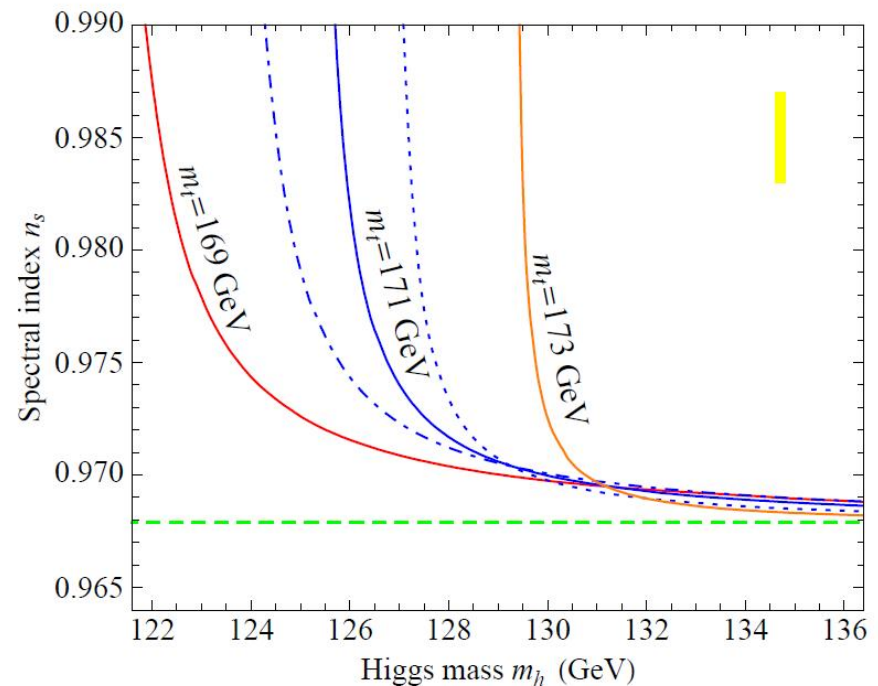
- Theory remains perturbative at high energy,
- Consistent inflation; consistent with WMAP!

# Top mass: why we care?

## - Higgs-inflation

Bezrukov, Shaposhnikov '07-'08  
De Simone, Hertzberg, Wilczek'08

Provided it works ☺  
the model is very predictive!



De Simone, Hertzberg, Wilczek arXiv:0812.4946v2

**Figure 1:** The spectral index  $n_s$  as a function of the Higgs mass  $m_h$  for a range of light Higgs masses. The 3 curves correspond to 3 different values of the top mass:  $m_t = 169$  GeV (red curve),  $m_t = 171$  GeV (blue curve), and  $m_t = 173$  GeV (orange curve). The solid curves are for  $\alpha_s(m_Z) = 0.1176$ , while for  $m_t = 171$  GeV (blue curve) we have also indicated the 2-sigma spread in  $\alpha_s(m_Z) = 0.1176 \pm 0.0020$ , where the dotted (dot-dashed) curve corresponds to smaller (larger)  $\alpha_s$ . The horizontal dashed green curve, with  $n_s \simeq 0.968$ , is the classical result. The yellow rectangle indicates the expected accuracy of PLANCK in measuring  $n_s$  ( $\Delta n_s \approx 0.004$ ) and the LHC in measuring  $m_h$  ( $\Delta m_h \approx 0.2$  GeV). In this plot we have set  $N_e = 60$ .

# Top mass: why we care?

So, to summarize, the top mass is needed:

- ❖ with numerical precision,
- ❖ with confidence about its definition.

Recall: mass is not observable; it is a formal parameter and is thus sensitive to its formal definition.

Unless we have a reasonable control over both mass definition and mass value, we cannot be confident we are doing a good job!

# How to measure the top mass?

At the LHC the top mass measurement can be done with “confidence”

Here is the idea:

- Find an observable sensitive to the value of the top mass;
- Fix all other parameters and fit the data by tuning the mass.  
(of course, we hope for data with sufficient statistics 😊 )
- If beyond LO, we become sensitive to the definition of the mass, too.

Example 1: the total top-pair cross-section.

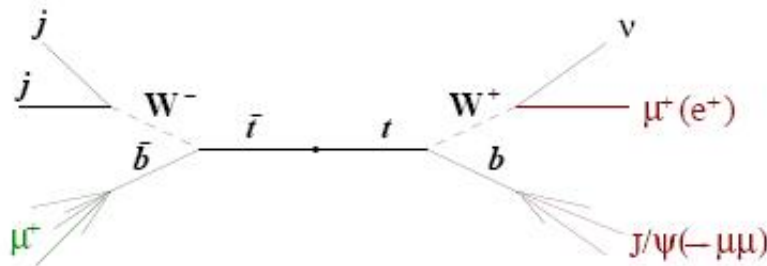
- It allows extraction of the mass with  $\sim 4\%$  accuracy.

Hint: compare to the current best value from the Tevatron  $\sim 0.8\%$

It is not all bad news: we are confident about what we measure 😊

# Example 2: "J/Psi final state"

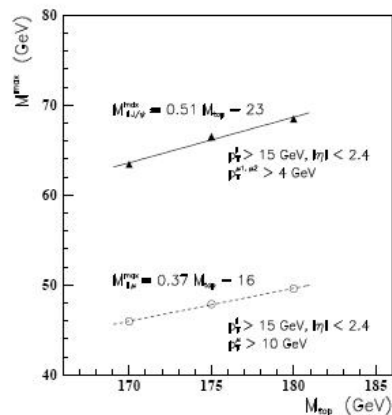
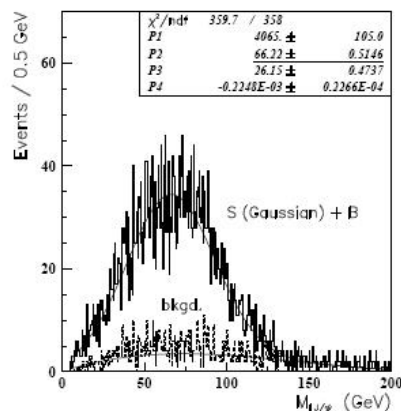
Jet measurements are hard at the LHC; check out the lepton signal



Proposed by: A. Kharchilava '99

R. Chierici, A. Dierlamm CMS NOTE 2006/058  
Corcella, Mangano, Seymour '00

**Idea:** - study the invariant mass distribution of  $M_{J/\psi-\ell}$  in top decay  
- explore the strong correlation between peak position and  $M_{\text{top}}$



Experimentally very clean signal  
Low branching ratio  $\sim 10^{-5}$ , but  
Compensated by large top rates  
 $\sim 1000$  events/year at LHC (14 TeV)  
Accuracy  $\leq 1$  GeV achievable.

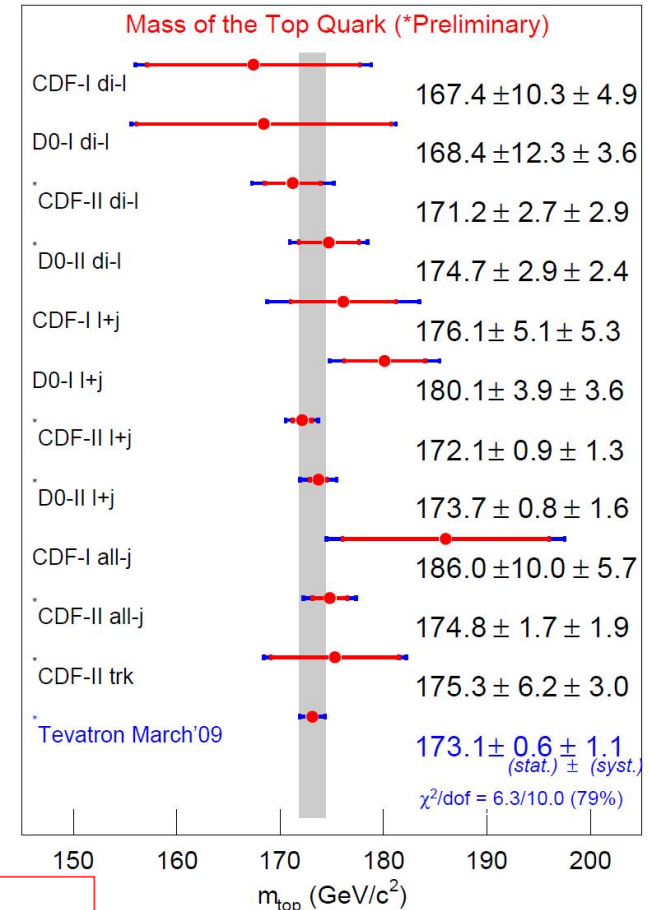
# The Tevatron: the latest numbers

A combination of 11 measurements: CDF+D0: 0903.2503

	Run I published					Run II preliminary					
	CDF			DØ		CDF				DØ	
	all-j	l+j	di-l	l+j	di-l	l+j	di-l	all-j	trk	l+j	di-l
$f \mathcal{L} dt$	0.1	0.1	0.1	0.1	0.1	3.2	1.9	2.9	1.9	3.6	3.6
Result	186.00	176.10	167.40	180.10	168.40	172.14	171.15	174.80	175.30	173.75	174.66
iJES	0.00	0.00	0.00	0.00	0.00	0.74	0.00	1.64	0.00	0.47	0.00
aJES	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.91	1.32
bJES	0.60	0.60	0.80	0.71	0.71	0.38	0.40	0.21	0.00	0.07	0.26
cJES	3.00	2.70	2.60	2.00	2.00	0.32	1.73	0.49	0.60	0.00	0.00
dJES	0.30	0.70	0.60	0.00	0.00	0.08	0.09	0.08	0.00	0.84	1.46
rJES	4.00	3.35	2.65	2.53	1.12	0.40	1.90	0.21	0.10	0.00	0.00
lepPt	0.00	0.00	0.00	0.00	0.00	0.18	0.10	0.00	1.10	0.18	0.32
Signal	1.80	2.60	2.80	1.11	1.80	0.34	0.78	0.23	1.60	0.45	0.65
MC	0.80	0.10	0.60	0.00	0.00	0.51	0.90	0.31	0.60	0.58	1.00
UN/MI	0.00	0.00	0.00	1.30	1.30	0.00	0.00	0.00	0.00	0.00	0.00
BG	1.70	1.30	0.30	1.00	1.10	0.50	0.38	0.35	1.60	0.08	0.08
Fit	0.60	0.00	0.70	0.58	1.14	0.16	0.60	0.67	1.40	0.21	0.51
CR	0.00	0.00	0.00	0.00	0.00	0.41	0.40	0.41	0.40	0.40	0.40
MHI	0.00	0.00	0.00	0.00	0.00	0.09	0.20	0.17	0.70	0.05	0.00
Syst.	5.71	5.28	4.85	3.89	3.63	1.35	2.98	1.99	3.11	1.60	2.43
Stat.	10.00	5.10	10.30	3.60	12.30	0.94	2.67	1.70	6.20	0.83	2.92
Total	11.51	7.34	11.39	5.30	12.83	1.64	4.00	2.61	6.94	1.80	3.80

Parameter	Value ( $\text{GeV}/c^2$ )
$M_t^{\text{all-j}}$	$175.1 \pm 2.6$
$M_t^{\text{l+j}}$	$172.7 \pm 1.3$
$M_t^{\text{di-l}}$	$171.4 \pm 2.7$

Signal includes:  
Theory and pdf uncertainties.  
Seems smallish.



# The Tevatron

- ❖ “Best” channel: lepton + jet.

- ❖ Relatively few top-pair events:

For example the latest published sample in the (lepton+jet) includes  $\sim 220$  events!

- ❖ This is not exactly big statistics (in the usual sense);

So, how is such precise extraction possible?

*Matrix element methods*

# Back to top physics at the Tevatron

## References:

Kondo et al: late 80's mid 90's

Dalitz and Goldstein: 90's

See also Adam Gibson, PhD Thesis, '06  
FERMILAB-PUB-08-242-E

NOTE: in the following I'll consider only the (lepton+jet) mode!

Experimentalist study events with:

1 lepton + (**exactly**) 4jets + large missing  $E_T$

At least one jet is required to be tagged as b-jet.

# Back to top physics at the Tevatron

## Step 1:

Here I follow *arXiv:hep-ph/9802249v1*

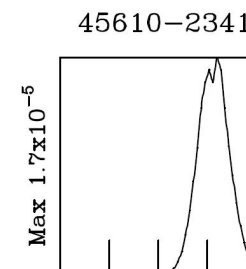
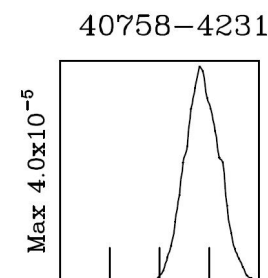
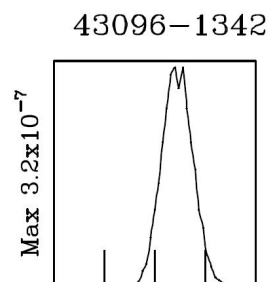
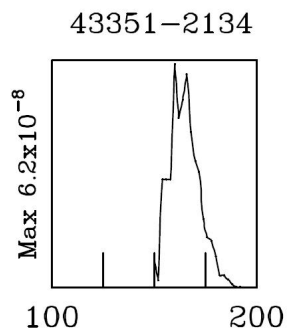
*Take the measured configuration of momenta for the final leptons and jets in a single event  $i$  and evaluate the probability*

$$P_i(m) = P(\text{configuration event } i | m)$$

*that these production and decay processes could produce the observed configuration if the top quark mass were  $m$ .*

Hint: calculated as from LO QCD

Examples of  $P_i(m)$  for few Tevatron events:



# The essence of the procedure

## Step 2:

apply Bayes' Theorem:

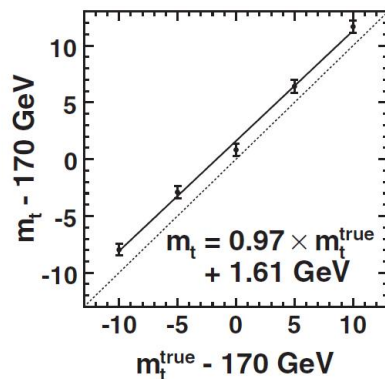
$$P(m|\text{data set } \{i\}) = \prod_{i=1}^N P(\text{event } i|m) \cdot \Phi(m)$$

a priori probability that the top mass is  $m$

In practice, what one does is:

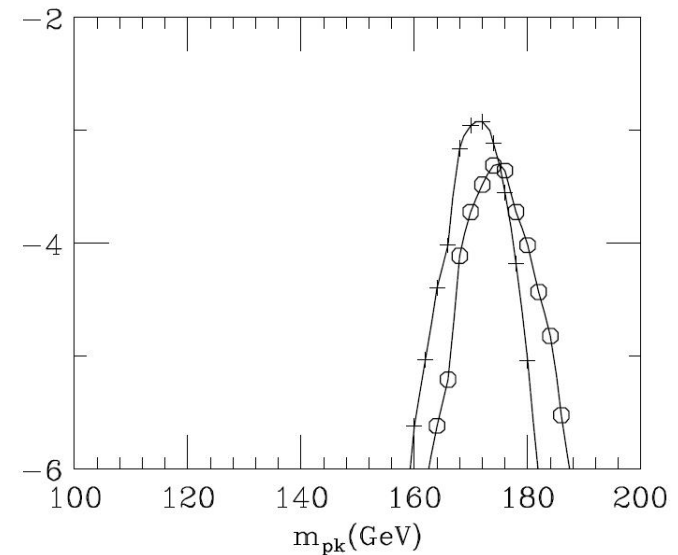
✓ Construct  $P(m) = \prod_{i=1}^N P_i(m)$

✓ Infer  $m_{\text{top}}$  from its extremum:



FERMILAB-PUB-08-242-E

$\log_{10}(\text{joint probability})$



arXiv:hep-ph/9802249v1

# ... and the complete procedure

FERMILAB-PUB-08-242-E

**1)**  $P(\text{event } i|m) = A(x)[fP_{\text{sig}}(x; m_t, k_{\text{jes}}) + (1 - f)P_{\text{bkg}}(x; k_{\text{jes}})]$

Signal fraction

LO t-tbar

LO W+4jets

**2)**  $P_{\text{sig}} = \frac{1}{N} \int \sum d\sigma(y; m_t) dq_1 dq_2 f(q_1) f(q_2) W(y, x; k_{\text{jes}})$

PDF's

Parton  $\rightarrow$  hadron  
+ detector resolution

**3)** Construct likelihood function:  $L(x; m_t, k_{\text{jes}}, f) = \prod_{i=1}^N P(\text{event } i|m)$

**4)** Extract " f " by minimizing  $-\ln L$

**5)** Finally construct  $L(x; m_t) = \int L(x; m_t, k_{\text{jes}}) G(k_{\text{jes}}) d\tilde{k}_{\text{jes}}$

**6)** Extract " m " from its maximum.

Prior probability;  
a Gaussian

# Comments

- ❖ At the Tevatron the statistics is small for standard analyses: Bayesian approach developed and applied (pretty solid 😊 )

- ❖ The procedure assumes we know exactly the distributions

  - for calibrations,

  - and for calculation of per event probabilities.

But that is not so: NLO brings 50% corrections => that is large uncertainty.  
How does that affect the extraction?

- ❖ Has this been studied?

- ❖ For theorists: even if the above is implemented at NLO, we do not have complete top-pair production and decay at NLO!

- ❖ And it must be fast!

# Forward-Backward Asymmetry

# Forward-Backward Asymmetry

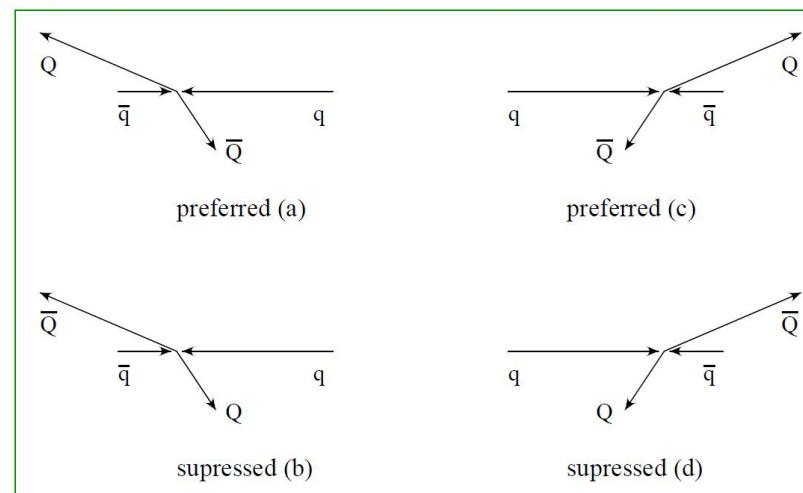
t-tbar: Kuhn, Rodrigo '98

- ✓ LO QCD: 0 asymmetry
- ✓ NLO QCD:  $0.05 \pm 0.015$
- ✓ CDF '08:  $A_{fb}^{p\bar{p}} = 0.17 \pm 0.08$
- ✓ CDF Note 9724 '09:

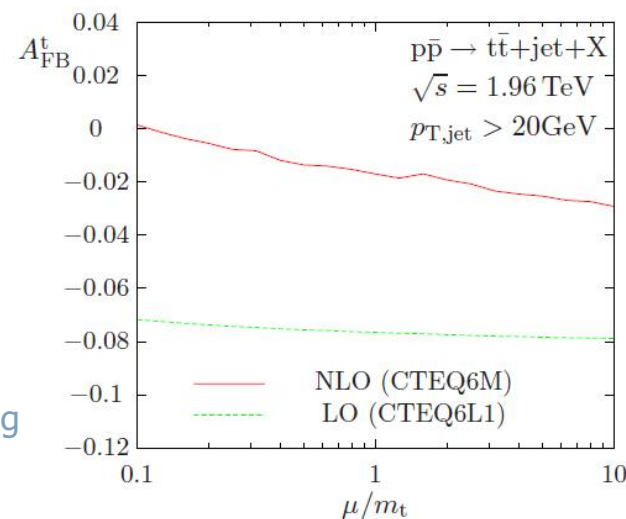
$$A_{fb} = 0.193 \pm 0.065^{stat} \pm 0.024^{syst}$$

Looks like  $2\sigma$  deviation

- ❖ BSM explanations not easy  
talk by J. Wells Top 09 workshop CERN
- ❖ QCD higher order effects?
  - soft gluon resum. small. Almeida, Sterman, Vogelsang
  - hard NNLO emissions could be large.
  - PDF's?



Talk by J. Kuhn, Top 09 workshop CERN



→ t-tbar+jet: Dittmaier, Uwer, Weinzierl '07

# Summary and Conclusions

Theorists and experimentalists should talk more to each other 😊

# Backup Slides

# Backup: Construct the Cross-section

How we put all this to work?

- ❖ Match fixed order and resummed results:

$$\sigma_{\text{RESUM}} = \sigma_{\text{NLO}} + \sigma_{\text{SUDAKOV}} - \sigma_{\text{OVERLAP}}$$

Known at NLO, not at NNLO

- ❖  $\sigma_{\text{NLO}}$  is known exactly,

- ❖  $\sigma_{\text{SUDAKOV}}$  : anomalous dimensions and matching coefficients needed.

$$\sigma_{ij}^{\text{TOT}}(N) = \sigma_{ij,1}(N) + \sigma_{ij,8}(N)$$

Known at NLO M. Czakon, A.M. '08

$$\sigma_{ij,I}(N) = \sigma_{ij,I}^{\text{Born}}(N) \sigma_{ij,I}^{\text{H}} \Delta_{ij,I}(N)$$

# Backup: Numerical Findings at NLO

$C_3$  numerics: -5%,  
color singlet channel: -12%,  
color octet channel: -3%,

$$\begin{aligned}\sigma_{gg}^{H(\text{BCMN})} &= 1 + \frac{\alpha_s}{\pi} 14.39 + o(\alpha_s^2), \\ \sigma_{gg}^{H(\text{BCMN})}|_{C_3 \text{ exact}} &= 1 + \frac{\alpha_s}{\pi} 12.04 + o(\alpha_s^2), \\ \sigma_{gg,1}^H &= 1 + \frac{\alpha_s}{\pi} 9.16 + o(\alpha_s^2), \\ \sigma_{gg,8}^H &= 1 + \frac{\alpha_s}{\pi} 13.19 + o(\alpha_s^2),\end{aligned}$$

Their implications :

- ✓ Formally these effects are beyond NLL; yet significant numerically
- ✓ Must be taken into account beyond NLL !

# Backup: Some statistics:

The answer involves not-so-popular statistical methods:

REF: PDG '08 - Statistics  
- Probability

❖ Frequentist statistics (the usual one):

*probability is interpreted as the frequency of the outcome of a repeatable experiment.*

❖ Bayesian statistics:

*the interpretation of probability is more general and includes degree of belief (called subjective probability). One can then speak of a probability density function (p.d.f.) for a parameter, which expresses one's state of knowledge about where its true value lies.*

Bayes' theorem

$$P(\text{theory}|\text{data}) \propto P(\text{data}|\text{theory})P(\text{theory})$$

**Interpretation:** *the prior degree of belief is updated by the data from the experiment*

Proof:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

# Backup: More Statistics

REF: PDG '08 - Statistics

p.d.f. := probability density function

*In Bayesian statistics, all knowledge about  $\theta$  is summarized by the **posterior** p.d.f.  $p(\theta|x)$ , which gives the degree of belief for  $\theta$  to take on values in a certain region given the data  $x$ .*

$$p(\theta|x) = \frac{L(x|\theta)\pi(\theta)}{\int L(x|\theta')\pi(\theta') d\theta'}$$

$L(x|\theta)$  - the likelihood function, i.e., the joint p.d.f. for the data given a certain value of  $\theta$ ,  
 $\pi(\theta)$  - the prior p.d.f. for  $\theta$ .

*Bayesian statistics supplies no unique rule for determining  $\pi(\theta)$ ; this reflects the experimenter's subjective degree of belief about  $\theta$  **before** the measurement was carried out*

# Backup: The method of maximum likelihood

*How to get L?*

$$p(\boldsymbol{\theta}|\boldsymbol{x}) = \frac{L(\boldsymbol{x}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})}{\int L(\boldsymbol{x}|\boldsymbol{\theta}')\pi(\boldsymbol{\theta}') d\boldsymbol{\theta}'}$$

*REF: PDG '08 - Statistics*

## The method of maximum likelihood

*Suppose we have a set of  $N$  measured quantities  $\boldsymbol{x} = (x_1, \dots, x_N)$  described by a joint p.d.f.  $f(\boldsymbol{x}; \boldsymbol{\theta})$ , where  $\boldsymbol{\theta} = (\theta_1, \dots, \theta_n)$  is set of  $n$  parameters whose values are unknown.*

*The likelihood function is given by the p.d.f. evaluated with the data  $\boldsymbol{x}$ , but viewed as a function of the parameters, i.e.,  $L(\boldsymbol{\theta}) = f(\boldsymbol{x}; \boldsymbol{\theta})$ .*

*If the measurements  $x_i$  are statistically independent and each follow the p.d.f.  $f(x; \boldsymbol{\theta})$ , then the joint p.d.f. for  $\boldsymbol{x}$  factorizes and the likelihood function is:*

$$L(\boldsymbol{\theta}) = \prod_{i=1}^N f(x_i; \boldsymbol{\theta}) \quad \text{Then:} \quad \frac{\partial \ln L}{\partial \theta_i} = 0$$

*Gives the maximum likelihood estimators, i.e.  $\boldsymbol{\theta} = (\theta_1, \dots, \theta_n)$*

**Hint:**  $\boldsymbol{\theta}$  – is to be  $m_{top}$